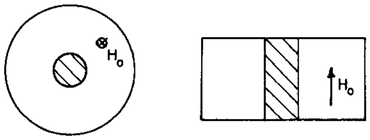

Fig. 4. Waveguides with combined symmetries. (a) Circular waveguide with axially magnetized ferrite rod (reflection and inversion symmetries). (b) Rectangular waveguide with transversely magnetized ferrite slab (180° rotation and inversion symmetries).

then the modal electromagnetic field components must be either even functions of both x and y or odd functions of both x and y. If the waveguide possesses both 180° rotation and inversion symmetry [see Fig. 4(b) for an example], then the modal electromagnetic field components must be either even or odd functions of x (the 180° rotation axis); there are no restrictions on the y variation.

III. Conclusions

It has been shown that a waveguide containing gyrotropic media with a biasing magnetic field H_0 will be bidirectional if it possesses one of the following symmetries: reflection symmetry in a plane perpendicular to the waveguide axis with H_0 parallel to the waveguide axis; 180° rotation symmetry about an axis perpendicular to the waveguide axis with H_0 parallel to the rotation axis; or inversion in a point on the waveguide axis with no restriction on the direction of H_0. It is conjectured that the possession of one of these symmetries is also a necessary condition for a waveguide containing gyrotropic media to be bidirectional, but this has not been proved.

In the examples shown in Figs. 1–4, the biasing magnetic field is uniform in direction and magnitude over the waveguide cross section. The relationship between bidirectionality and the symmetry conditions holds more generally for cases where the direction or magnitude of the biasing magnetic field varies over the cross section. However, any such variation must be properly accounted for in the determination of the symmetry properties of the gyrotropic media in the waveguide.

A comment concerning the relation between reciprocity and bidirectionality is warranted. An arbitrary current excitation at plane z_1 of a waveguide will produce a related electromagnetic field at plane z_2. If this same current excitation, when located at plane z_2, produces an identical electromagnetic field at plane z_1, then the waveguide is said to be reciprocal. A bidirectional waveguide containing gyrotropic media will, in general, not exhibit this reciprocity. For example, suppose a linearly polarized current excites the dominant modes of the bidirectional waveguide shown in Fig. 4(a) (the dominant modes are related to the H_{11} modes of an empty circular waveguide). This excitation will produce an electromagnetic field which exhibits Faraday rotation [7], which is a nonreciprocal effect. On the other hand, if the current excitation is such as to excite only those modes of this waveguide which are related to the E_{0n} modes of an empty circular waveguide, reciprocal behavior will be observed [8]. The symmetry conditions establish whether a waveguide containing gyrotropic media will be bidirectional, but they do not determine whether or not reciprocal behavior might be observed for some particular excitation.

Appendix

The transformation properties of electric fields (polar vectors) and magnetic fields (axial vectors) under reflection, 180° rotation, and inversion are summarized here. For a more complete discussion of the properties of polar and axial vectors see [9] or [10]; [11] presents an excellent comparison of the transformation properties of electric and magnetic fields based on physical arguments. Using primes to label the transformed field components, reflection of electric and magnetic fields in the $z = 0$ plane yields

$$\begin{aligned} \boldsymbol{E}_T'(x,y,z) &= \boldsymbol{E}_T(x,y,-z) \qquad \boldsymbol{H}_T'(x,y,z) = -\boldsymbol{H}_T(x,y,-z) \\ E_z'(x,y,z) &= -E_z(x,y,-z) \qquad H_z'(x,y,z) = H_z(x,y,-z). \end{aligned} \tag{8}$$

Rotation of electric and magnetic fields by 180° about the x axis yields

$$\begin{aligned} E_x'(x,y,z) &= E_x(x,-y,-z) \qquad H_x'(x,y,z) = H_x(x,-y,-z) \\ E_y'(x,y,z) &= -E_y(x,-y,-z) \qquad H_y'(x,y,z) = -H_y(x,-y,-z) \\ E_z'(x,y,z) &= -E_z(x,-y,-z) \qquad H_z'(x,y,z) = -H_z(x,-y,-z). \end{aligned} \tag{9}$$

Inversion of electric and magnetic fields in $z = 0$ yields

$$\boldsymbol{E}'(x,y,z) = -\boldsymbol{E}(-x,-y,-z) \qquad \boldsymbol{H}'(x,y,z) = \boldsymbol{H}(-x,-y,-z). \tag{10}$$

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The Piezoelectric-Magnetoelastic Wave Propagation Through the Conducting Plate in a Composite Medium

TUSHAR BHATTACHARYYA,
MAKOTO TSUTSUMI, MEMBER, IEEE, AND
NOBUAKI KUMAGAI, SENIOR MEMBER, IEEE

Abstract—**The piezoelectric and magnetoelastic surface wave propagation through a composite layered structure of one piezoelectric and another magnetoelastic media is considered with a metal plate placed in between them. The dispersion relations have been derived and numerically computed. Thereafter, the field distributions are evaluated.**

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T. Bhattacharyya is with the Faculty of Engineering, Osaka University, Suita-shi, Japan, on leave from Jadavpur University, Calcutta, India.
M. Tsutsumi and N. Kumagai are with the Faculty of Engineering, Osaka University, Suita-shi, Japan.

These results may find usefulness for the realization of nonreciprocal acoustic surface wave devices.

I. Introduction

The present trend of research in investigating the behaviors of surface waves in piezoelectric and magnetoelastic materials led to the choice of the problem being discussed in this short paper. Many research reports have already been published on this subject [1]–[3]. Of these, the investigations which need special attention are the existence of shear-type elastic surface waves along a surface of piezoelectric material as shown by Bleustein [4]. Also Matthews *et al.* and Van de Vaart investigated the case of shear-type surface wave propagation on the surface of ferrimagnetic materials [3], [5]. On the other hand, Maerfeld and Tournois established the presence of an acoustic shear surface wave guided by the interface of two semi-infinite media in contact, if one, at least, of these two media is piezoelectric [6].

In the framework of the present short paper, the shear-type piezoelectric and magnetoelastic surface wave propagation through a composite layered structure of one piezoelectric (ZnO) and another magnetoelastic (GaYIG) medium is being considered and a metal plate of any arbitrary finite thickness is assumed to be inserted between the two media. The relevant dispersion relation has been derived and it has been shown that such structure does not possess the same propagation characteristics in both $+y$ and $-y$ directions of propagation. Later, the case of a thin metal plate (with negligible thickness) is treated and the necessary expressions for the field distributions have been obtained to know the concentration of power near the surface.

II. Dispersion Relation

The geometry of the structure considered is shown in Fig. 1. The configuration considers a thick metal plate of thickness h inserted in between the media of ZnO of crystal class 6 mm and GaYIG. The biasing magnetic field and the polarization axis in ZnO are assumed to be applied in Z direction. Based on the assumption of nondependence of the field on the Z axis and the variation of the field $e^{j\omega t}$ to be implicit, the equation of motion for shear-type piezoelectric wave and the electric potential ϕ_E for the piezoelectric medium can be written from Maxwell's equation as [4]

$$-\omega^2 \rho_E R_{ZE} = \bar{C}_{44}{}^E \underline{\nabla}^2 R_{ZE} \tag{1}$$

and

$$\underline{\nabla}^2 \psi_E = 0 \tag{2}$$

where

$$\bar{C}_{44}{}^E = C_{44}{}^E + \frac{e_{15}{}^2}{\varepsilon_E}.$$

In the preceding equations, $C_{44}{}^E$ is the elastic constant of the medium and R_{ZE} represents the elastic displacement in Z direction. Again the equation for electric potential function can be represented as [4]

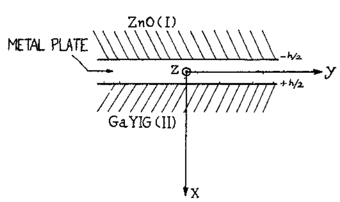


Fig. 1. Geometry of the composite structure.

$$\psi_E = \phi_E - \frac{e_{15}}{\varepsilon_E} R_{ZE} \tag{3}$$

where e_{15} is the piezoelectric constant. The symbols ρ_E and ε_E have the usual meanings of density and dielectric constant, respectively.

Following a similar procedure, the equation of motion for the shear-type magnetoelastic wave and the magnetic potential function ϕ_H in the magnetoelastic medium can be represented as [3], [5]

$$-\omega^2 \rho_H R_{ZH} = \bar{C}_{44}{}^H \underline{\nabla}^2 R_{ZH} \tag{4}$$

and

$$\underline{\nabla}^2 \psi_H = 0 \tag{5}$$

where

$$\bar{C}_{44}{}^H = C_{44}{}^H + \frac{(b\gamma)^2 \mu_0 H_i}{M(\omega^2 - \omega_0{}^2)}.$$

The term $C_{44}{}^H$ represents the elastic constant of the medium concerned and

$$\omega_0{}^2 = (\mu_0 \gamma)^2 H_i (H_i + M).$$

And also ψ_H is given by

$$\psi_H = \phi_H - \frac{\mu_0 H_i \gamma^2 b}{\omega^2 - \omega_0{}^2} R_{ZH} \tag{6}$$

where R_{ZH} is the displacement in the direction of the applied magnetic field. In the preceding sets of equations, the nomenclatures used are ρ_H, b, γ, M, and H_i which are density, the magnetoelastic coupling constant, the gyromagnetic ratio, the saturation magnetization, and the internal biasing magnetic field, respectively.

The resulting stress tensor components for the piezoelectric and magnetoelastic mediums can be obtained, respectively, as

$$T_{xZE} = \bar{C}_{44}{}^E \frac{\partial R_{ZE}}{\partial x} + e_{15} \frac{\partial \psi_E}{\partial x} \tag{7}$$

and

$$T_{xZH} = \bar{C}_{44}{}^H \frac{\partial R_{ZH}}{\partial x} + \frac{j\omega\gamma b^2}{M(\omega^2 - \omega_0{}^2)} \frac{\partial R_{ZH}}{\partial y} + \frac{\mu_0 \gamma b}{\omega^2 - (\gamma \mu_0 H_i)^2} \left[\mu_0 \gamma H_i \frac{\partial \psi_H}{\partial x} + j\omega \frac{\partial \psi_H}{\partial y} \right]. \tag{8}$$

The expression for the stress tensor T_{xz} within the metal can be written from (7) by considering $e_{15} = 0$ and assuming R_{ZE} to be equal to R_z and also the fact that $\bar{C}_{44}{}^E$ changes to the value of C_{44} where C_{44} is the stiffness constant of the metal. Thus

$$T_{xz} = C_{44} \frac{\partial R_z}{\partial x}. \tag{9}$$

The solutions of the preceding equations are assumed to have the following forms (from the boundary condition at $x = \pm\infty$):

$$R_{ZE} = \hat{R}_{ZE} e^{\xi x} e^{\mp j k_y y} \tag{10}$$

$$R_{ZH} = \hat{R}_{ZH} e^{-\eta x} e^{\mp j k_y y} \tag{11}$$

$$R_Z = (R_{Z1} \cos \nu x + R_{Z2} \sin \nu x) e^{\mp j k_y y} \tag{12}$$

$$\psi_E = \hat{\psi}_E e^{k_y x} e^{\mp j k_y y} \tag{13}$$

and

$$\psi_H = \hat{\psi}_H e^{-k_y x} e^{\mp j k_y y} \tag{14}$$

where

$$\xi = \sqrt{k_y^2 - \frac{\omega^2 \rho_E}{\bar{C}_{44}^E}}$$

$$\eta = \sqrt{k_y^2 - \frac{\omega^2 \rho_H}{\bar{C}_{44}^H}}$$

and

$$\nu = \sqrt{\frac{\omega^2 \rho'}{C_{44}} - k_y^2}.$$

The necessary and sufficient boundary conditions are that the normal component of the stress tensor and the displacement terms can be equated at the corresponding interfaces of the media and the metal plate. Also the electric potential vanishes at $x = -h/2$ and the normal components of magnetic flux density equal zero at $x = +h/2$. With the application of the above mentioned conditions, the expression for the dispersion characteristics becomes

$$-\frac{\tan \nu h}{C_{44}\nu} = \frac{\bar{C}_{44}^E\xi + \bar{C}_{44}^H\eta \mp \dfrac{\omega k_y \gamma b^2(\gamma\mu_0 H_i \pm \omega)}{M(\omega^2 - \omega_0^2)(\omega_m \pm \omega)} - \dfrac{k_y e_{15}^2}{\varepsilon_E}}{\eta\xi\bar{C}_{44}^E\bar{C}_{44}^H - C_{44}^2\nu^2 \pm \dfrac{\omega k_y \gamma b^2(\gamma\mu_0 H_i \pm \omega)}{M(\omega^2 - \omega_0^2)(\omega_m \pm \omega)}\left[\dfrac{k_y e_{15}^2}{\varepsilon_E} - \bar{C}_{44}^E\xi\right]}. \tag{15}$$

In (15)

$$\omega_m = \mu_0\gamma(H_i + M).$$

For numerical solution of (15), the assumed values of the material constants are the following: $\gamma = 1.76 \times 10^{11}$ $(\text{Wb/m}^2 \text{ s})^{-1}$, $\mu_0 H_i = 50 \times 10^{-4}$ Wb/m^2, $\mu_0 M = 300 \times 10^{-4}$ Wb/m^2, $b = 7.4 \times 10^5$ J/m^2, $e_{15} = -0.59$ C/m^2, $C_{44} = 2.85 \times 10^{10}$ N/m^2, $C_{44}^H = 7.64 \times 10^{10}$ N/m^2, $C_{44}^E = 4.25 \times 10^{10}$ N/m^2, $\rho = 1.93 \times 10^4$ kg/m^3, $\rho_H = 5.17 \times 10^3$ kg/m^3, $\rho_E = 5.68 \times 10^3$ kg/m^3, and $\varepsilon_E = 8.33 \times \varepsilon_0$ F/m. As a conducting plate, polycrystalline gold of class cubic m3m having thickness 1 μm is chosen.

To show clearly the effect of the magnetoelastic coupling, firstly, the dispersion characteristics in the absence of the magnetoelastic coupling is obtained simply by putting $b = 0$ in (15). Then it is numerically evaluated and plotted in Fig. 2(a) and represented by the solid line. The lines for $\eta = 0$, $\xi = 0$, and $\nu = 0$ mean, respectively, the volume mode of the magnetoelastic wave, the piezoelectric wave, and the elastic wave in the absence of the corresponding boundaries. As can be seen from Fig. 2(a), the dispersion curves lie in the region between $\xi = 0$ and $\nu = 0$ lines and thus yield two cutoff frequencies f_1 and f_2. Moreover, the dispersion curves are symmetric for both $\pm y$ directions and hence the propagation characteristics are reciprocal in nature—very similar to that of an ordinary shear elastic wave in a composite structure in the presence of the piezoelectric constant.

Next the dispersion curves are plotted; in Fig. 2(b) the case where magnetoelastic coupling is present (i.e., $b \neq 0$) is represented by the solid lines. The effect of the coupling between the magnetostatic wave and the ordinary shear elastic wave exhibits itself at particular frequencies like $f_0(\omega_0)$ and $f_m(\omega_m)$. The upper frequency f_m is due to the coupling between the elastic wave and the magnetostatic surface wave influenced by the existence of the metal plate where the split bandwidth, a measure of the coupling strength, depends on the metal thickness [5]. Also the propagation is allowed only in $-y$ direction because of the consideration of a semi-infinite medium [3], [7]. Thus it can be seen to be nonreciprocal in nature near the frequency $\omega = \omega_m$ ($f_m = \omega_m/2\pi = 964$ MHz). On the other hand, the lower coupling frequency f_0 represents the coupling between the elastic wave and the volume mode of the magnetostatic wave [7]. In this region, the propagation characteristics show the reciprocal nature in both $\pm y$ directions.

Hence the comparison between Fig. 2(a) and (b) reveals that the dispersion characteristics behave in the same way as the case where magnetoelastic coupling is absent excepting near the frequencies f_0 and f_m. Further, the dispersion relation for the thin metal plate case can be expressed conveniently, by assuming $h = 0$ in (15), as

$$\bar{C}_{44}^H \sqrt{1 - \frac{\rho_H}{\bar{C}_{44}^H}\left(\frac{\omega}{k_y}\right)^2} + \bar{C}_{44}^E \sqrt{1 - \frac{\rho_E}{\bar{C}_{44}^E}\left(\frac{\omega}{k_y}\right)^2} = \frac{e_{15}^2}{\varepsilon_E} \pm \frac{\gamma b^2 \omega(\omega \pm \mu_0\gamma H_i)}{M(\omega^2 - \omega_0^2)(\omega \pm \omega_m)}. \tag{16}$$

In this case, the mass effect of the metal plate can be neglected. In (16), the terms involved are explained already. The small dashed line in Fig. 2(b) represents the $\omega - k$ diagram for the thin metal plate case, using same material constants mentioned before. As can be seen, the solution appears in the $-y$ direction only and this becomes evident from the curve, where the frequency bandwidth near the frequency f_m is about 10 MHz with k value ranging between 1.9×10^6 and infinity.

III. Field Distribution in Thin Metal Plate Case

To know the concentration of power near the surface, which is important to investigate from physical aspects, the field distribution as a function of depth is evaluated. The field equations (10)–(14), as applied to the case of a thin metal plate, are normalized with respect to (10) with imposition of the necessary boundary conditions. Thus

$$R_{ZE} = \hat{R}_{ZE} e^{\xi x} e^{\mp jk_y y} \tag{17}$$

$$R_{ZH} = \hat{R}_{ZE} e^{-\eta x} e^{\mp jk_y y} \tag{18}$$

$$\phi_E = -\hat{R}_{ZE}\frac{e_{15}}{\varepsilon_E}(e^{k_y x} - e^{\xi x}) e^{\mp jk_y y} \tag{19}$$

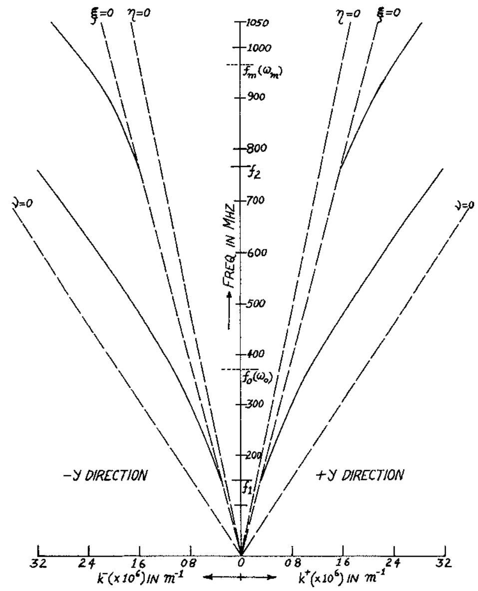
and

$$\phi_H = \hat{R}_{ZE}\frac{\gamma b}{\omega^2 - \omega_0^2}\left[\mu_0 H_i \gamma e^{-\eta x} \mp \frac{\omega(\omega \pm \gamma\mu_0 H_i)}{\omega \pm \omega_m} e^{-k_y x}\right] e^{\mp jk_y y}. \tag{20}$$

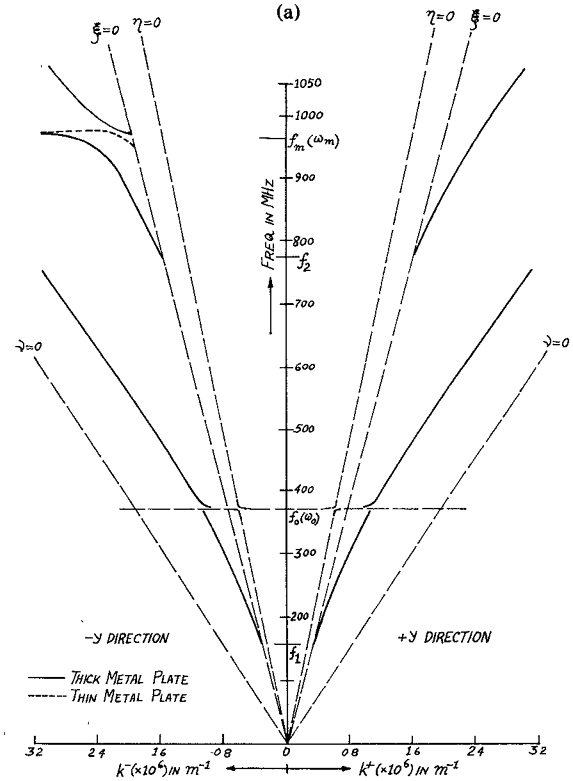
Fig. 3(a) represents the distribution of the potential function in depth direction and Fig. 3(b) explains the particle displacement as a function of depth in both $+x$ and $-x$ directions very near to ω_m. All the field components exhibit exponential decaying nature. Moreover, the characteristics of the elastic displacements are similar to the characteristics of Stonely wave case [1].

IV. Conclusive Discussions

In the adjoining figures, the dispersion diagrams for thick and thin metal plate cases are being shown. As can be seen from the ω–k characteristics for thick metal plate case, the propagation



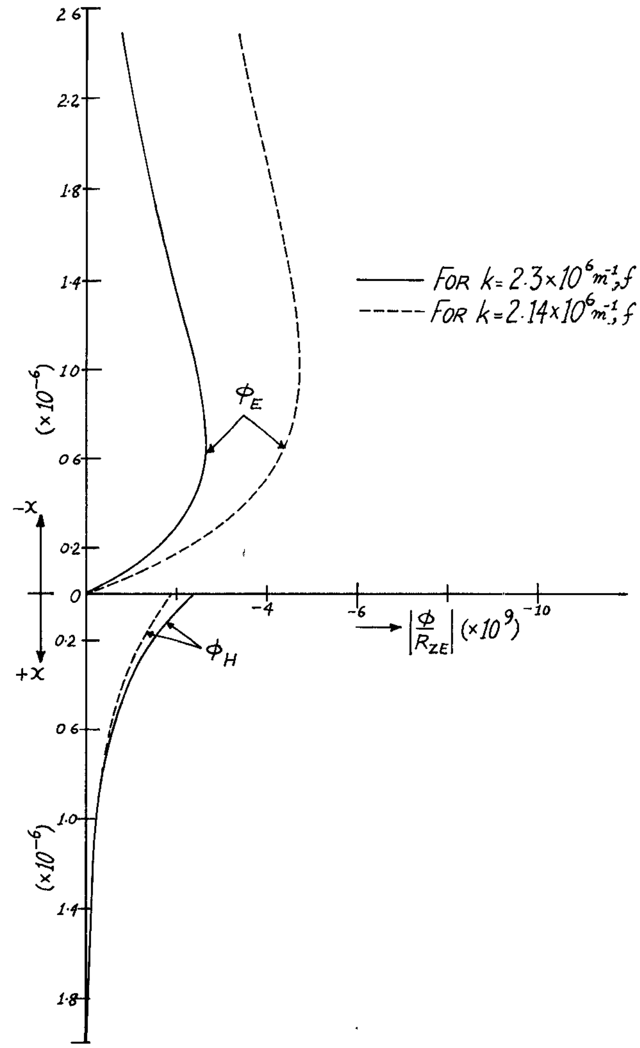
(a)



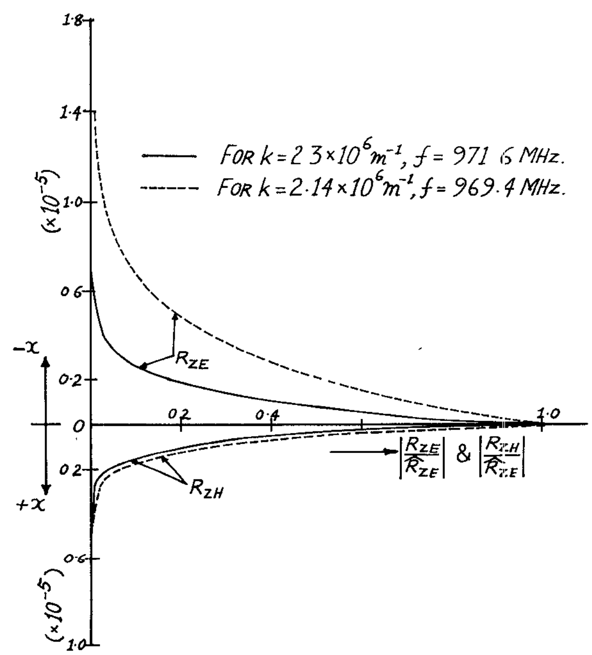
(b)

Fig. 2. (a) The dispersion diagram for $+y$ and $-y$ directions of propagation in the absence of the magnetoelastic coupling. (b) The dispersion diagram for $+y$ and $-y$ directions of propagation.

characteristics, around the frequency f_m, become nonreciprocal due to the magnetostatic surface wave. As a matter of fact, the forward wave still retains its monotonic increasing behavior due to the shear elastic wave of the elastic layered structure, but the backward wave does not. In contrast, the dispersion characteristics for a thin metal plate case are purely nonreciprocal in nature. Also the propagation characteristics are constrained in a rather narrow band. The aforesaid structure, being discussed in the present short paper, may find extensive usefulness in the realization of nonreciprocal surface wave devices and also for the excitation of magnetoelastic surface wave using a piezoelectric surface wave.



(a)



(b)

Fig. 3. ——: For $k = 2.3 \times 10^6\ \mathrm{m}^{-1}$, $f = 971.6$ MHz. ----: For $k = 2.14 \times 10^6\ \mathrm{m}^{-1}$, $f = 969.4$ MHz. (a) The distribution of potential function in depth direction. (b) The variation of particle displacement as a function of depth.

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